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Inverse kinematics of IGM welding robot by elimination approach

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A B S T R A C T : In this paper, the inverse kinematics of IGM welding robot is presented. The method used is elimination method, as it eliminates the joint variables one by one to get an equation in only one joint variable as unknown. The final equation is 16th degree polynomial which can be solved for getting one joint variable and rest of the variables can be found out by reversing the steps and re arranging the basic direct kinematics equation. The software programme is created in MATLAB to avoid symbolic and numerical errors.

Keywords: Inverse kinematics, elimination approach

1. Introduction

The direct kinematics problem is to find the position and orientation of the last link in the fixed or reference coordinate system for the given link parameters and the joint variables.

The inverse kinematics problem is to find the joint variables for the given link parameters and the position and orientation of the last link in the fixed or reference coordinate system.

L.W. Tsai and A.P. Morgan (1985)[0]showed that the problem can be reduced to that of solving a system of eight second-degree equations in eight unknowns. This second-degree system can be solved using a continuation algorithm. C. Wampler and A. Morgan (1991)[0] considered the computation of all solutions to the inverse position problem for general six-revolute-joint manipulators. Instead of reducing the problem to one highly complicated input-output equation, they worked with a system of very simple polynomial equations. Raghavan, M. and B. Roth (1993)[0] elaborated on a method developed by the authors for solving the inverse kinematics of a general 6R manipulator. The method is shown to yield a single polynomial, of minimum degree, in terms of just one of the joint variables. Manocha D. Canny J.F (1994)[0] expressed a polynomial as a matrix determinant and its roots are computed by reducing to an eigenvalue problem. The other roots of the multivariate system are obtained by computing eigenvectors and substitution.

2. Inverse kinematics

Manipulators consist of a group of rigid-bodies, or links, connected together by joints. The relative motion associated with each joint can be controlled such that the free-end, the hand, can be positioned in a desired

Nomenclature

a_i	length of link i + 1
αi	twist angle between the axes of joints i and i + 1
d_i	offset distance at joint i
θ_i	joint rotation angle at joint i

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manner. A manipulator has the form of an open-loop kinematic chain. Each link is connected to no more than two others, and the joints are either of the revolute or prismatic type.

The Denavit and Hartenberg notations are used as per nomenclatures in Tsai and Morgan(1985)0, the links of the 6R manipulator are numbered from 1 to 7, the fixed or base link being 1, and the outermost link or hand being 7. A coordinate system is attached to he each link to facilitate a mathematical description of the linkage and the relative arrangement of the links. The coordinate system attached to the ith link is numbered i. The 4 X 4 transformation matrix relating coordinate systems i + 1 and i is as follows:

$$\begin{bmatrix} \cos\theta i & -\cos\alpha i\cdot\sin\theta i & \sin\alpha i\sin\theta i & ai\cos\theta i \\ \sin\theta i & \cos\alpha i\cdot\sin\theta i & -\sin\alpha i\sin\theta i & ai\sin\theta i \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Where,

$$[A]_{jt=}\begin{bmatrix} \cos\theta i & -\sin\theta i & 0 & 0 \\ \sin\theta i & \cos\theta i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } [A]_{st=}\begin{bmatrix} \cos\theta i & 0 & 0 & ai \\ 0 & \cos\alpha i & -\sin\alpha i & 0 \\ 0 & \sin\alpha i & \cos\alpha i & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

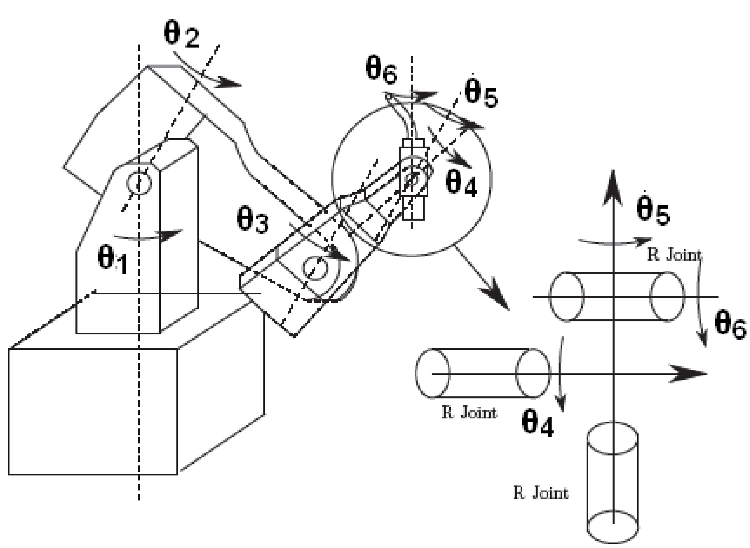

Fig. 1 IGM welding robot

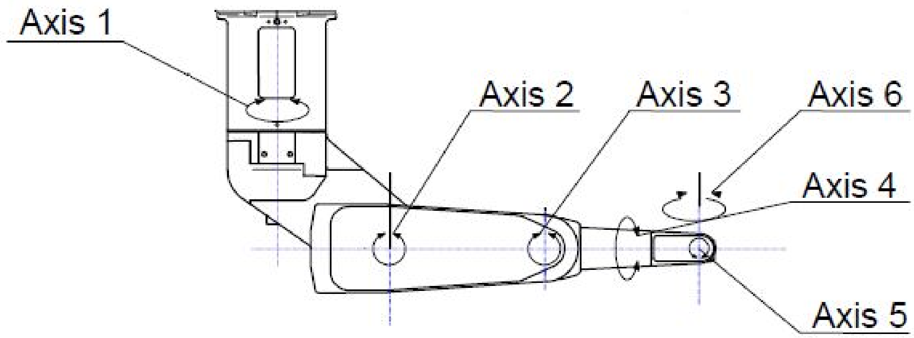

Fig. 2 IGM robot with 6R axes

3. Method of elimination

The closure equation for the 6R manipulator is the following matrix equation

$$A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 \cdot A6 = A_{hand} \qquad (2)$$

A_{hand} is the 4 X 4 transformation matrix describing the Cartesian coordinate system 7, attached to the hand or last link with respect to coordinate system 1, attached to the base link. The entries of this matrix are known because the hand coordinates in the goal position are specified. The left-hand side of the above matrix equation describes coordinate system 7 with respect to coordinate system 1, in terms of the relative arrangements of the intermediate coordinate systems. The quantities ai, di, i=1,..,6, appearing in the matrices on the left-hand side of Eq.2 are all known. The unknown quantities are θi, i=1,..,6, and the above matrix equation must be solved for them.

Let,

$$A_{hand} = \begin{bmatrix} lx & mx & nx & qx \\ ly & my & ny & qy \\ lz & mz & nz & qz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where, $C_i = \cos\theta i$, $S_i = \sin\theta i$ and i=1, 2, 3,4,5,6

$$lx = C6(C5(C4(C1C2C3 - C1S1S3) - S1S4) - S5(C1C2S3 + C1C3S2)) - S6(S4(C1C2C3 - C1S2S3) - C4S1)$$

$$mx = -C6(S4(C1C2C3 - C1S1S3) - C4S1) - S6(C5(C4(C1C2C3 - C1S2S3) - S1S4) - S5(C1C2S3 + C1C3S2))$$

$$nx = -S5(C4(C1C2C3 - C1S2S3) - S1S4) - C5(C1C2S3 + C1C3S2)$$

$$qx = a3(C1C2C3 - C1S2S3) - d4(C1C2S3 + C1C3S2) - d3S1 + d5(S4(C1C2C3 - C1S2S3) - C4S1) + a2C1C2$$

$$ly = C6(C5(C4(C2C3S1 - S1S2S3) + C1S4) - S5(C2S1S3 + C3S1S2)) - S6(C1C4 + S4(C2C3S1 - S1S2S3))$$

$$my = -C6(C1C4 + S4(C2C3S1 - S1S2S3)) - S6(C5(C4(C2C3S1 - S1S2S3) + C1S4) - S5(C2S1S3 + C3S1S2))$$

$$ny = -S5(C4(C2C3S1 - S1S2S3) + C1S4) - C5(C2S1S3 + C3S1S2)$$

$$qy = a3C1 + a3(C2C3S1 - S1S2S3) - d4(C2S1S3 + C3S1S2) + d5(C1C4 + S4(C2C3S1 - S1S2S3)) + a2C2S1$$

$$lz = S6(C2C4 + C3S2S4) - C6(C5(C2S4 + C3C4S2) - S2S3S5)$$

$$mz = C6(C2C4 + C3S2S4) + S6(C5(C2S4 + C3C4S2) - S2S3S5)$$

$$nz = S5(C2S4 + C3C4S2) + C5S2S3$$

$$qz = d4S2S3 - a2S2 - d5(C2C4 + C3S2S4) - a3C3S2 - d3C3$$

Now, from the above direct kinematic matrix, inverse kinematic solutions can be obtained by elimination method explained later.
The elimination method uses approach of Raghavan and Roth
Main steps of the method are:
(i) The homogeneous matrix transformation equation of a 6 degree of freedom serial manipulator

[0A6]= [0A1] [1A2][2A3][3A4][4A5][5A6] is written under the form:

([2A3] jt) [3A4][4A5] ([5A6])st=([2A3]st)-1[1A2]-1[0A1]-1[0A6]([5A6])jt-1 (1)

Where,

$$[A]_{jt} = \begin{pmatrix} \cos\theta i & -\sin\theta i & 0 & 0 \\ \sin\theta i & \cos\theta i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } [A]_{st} = \begin{pmatrix} \cos\theta i & 0 & 0 & ai \\ 0 & \cos\alpha i & -\sin\alpha i & 0 \\ 0 & \sin\alpha i & \cos\alpha i & di \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The third and fourth columns of above matrix equation are devoid of θ6. The third column is considered as p and fourth one is called l.
(ii) The following 14 scalar equations which are devoid of θ6 are calculated analytically:
Three equations from p.
Three equations from l.
One scalar equation from the scalar dot product p·p
One scalar equation from the scalar dot product p· l
Three equations from the vector cross product p×l
Three scalar equations from (p·p)l−(2p· l)p.
Where, p and l are the vectors formed by the first three elements of columns 3 and 4 respectively of equation (2))
(iii) These equations form the system:

MX=NY (3)

Where M is a 14 x 9 matrix and N is a 14 X 8 matrix. Every element in matrix M is a quadratic in x3 = tan (θ3/2) while every element in N is constant.
The vectors X, and Y for the 6R manipulator are equal to:
X = [S4S5, S4C5, C4S5, C4C5, S4, C4, S5, C5, 1]T
And Y = [S1S2, S1C2, C1S2, C1C2, S1,C1, S2, C2]T
With Si =sin (θi) and Ci =cos (θi)
Every element of the vectors X and Y is called a "power product" of equations (3).
(iv) 8 of the 14 equations are used to eliminate Y; the resulting system of 6 equations takes the form:

∑iXi=0 (4)

Where Xi is a 6 x 9 matrix.
The following substitution sin(θi)=2x/(1+xi) and cos(θi) = (1-xi2)/(1+xi2), where i = 3,4,5 and xi = tan (θi/2), lead to the system form:

∑iXi= 0 (5)

where Xi=[x24x25 , x24x5, x24,x4x25, x4x5, x4 ,x25, x5 , 1]'.
(vi) The 6 equations of (5) are multiplied by X4 and the following 12 x 12 homogeneous system is obtained:

∑iXi = 0 (6)

Where:
Xi =[x34x25, x34x5 , x34, x24x25 , x24x5, x24,x4x25, x4x5, x4 ,x25, x5 , 1]
(vii) The condition det∑ = 0 gives the characteristic polynomial of the manipulator in x3. For a 6R general manipulator the degree of this polynomial is 16.
(viii) We back substitute each real solution of θ3 in step vi and calculate a unique value for θ4 and θ5 from the solution of a linear system. Further substitution of θ3, θ4 and θ5 in step iii gives a unique value for θ1 ,θ2.
Finally, θ6 follows from two elements of the first two columns of the closure equation.
These two equations can be solved to get θ6.

4. Manipulator with non intersecting wrist

The three-axes-intersecting wrist, as in the PUMA manipulator, is a fairly complicated design and also difficult to manufacture. In addition, there is always some manufacturing tolerance that makes it impossible to have three axes intersecting at a point. Fig. **1** shows a six-DOF robot manipulator manufactured by IGM and often used for robotic welding. In this robot, as shown in Fig. **1** , the last three axes do not intersect and there is an offset d5. The rest of the Denavit - Hartenberg parameters are the same as in a PUMA. The Fig. shows the six axes of the robot. The D-H parameters are as per the **Error! Reference source not found.**.
The hand matrix for IGM robot for given D-H parameters

$$A_{hand} = \begin{bmatrix} 0.16425 & 0.82616 & 0.53882 & -0.033173 \\ 0.97484 & -0.21919 & 0.038834 & 0.1681 \\ 0.1502 & 0.51893 & -0.84146 & -0.78888 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The 14 equations obtained in above method are as follows
Eq1: C2*(qx*C1 + qy*S1) - qz*S2 =a2 + C3*(a3 - d5*S4) + d4*S3

Eq2: qz*C2 + S2*(qx*C1 + qy*S1) =D4*C3 – S3*(a3 - d5*S4),

Eq3 :qy*C1 - qx*S1=d3 + d5*C4

Eq4: C2*(nx*C1 + ny*S1) - nz*S2 =C5*S3 + C3*C4*S5, Eq5 : nz*C2 + S2*(nx*C1 + ny*S1)=C3*C5 – C4*S3*S5,

Eq6: ny*C1 - nx*S1 =S4*S5,

Eq7 : qx2 + qy2 + qz2=a22 + 2*C3*a2*a3 + 2*S3*a2*d4 - 2*C3*S4*a2*d5 + a32 - 2*S4*a3*d5 + d32 + 2*C4*d3*d5 + d42 + d52

Eq8: nx*qx + ny*qy + nz*qz =d4*C5 + a2*C5*S3 + a3*C4*S5 + d3*S4*S5 + a2*C3*C4*S5,

Eq9: ny*qx*S2 - nx*qy*S2 + ny*qz*C1*C2 - nz*qy*C1*C2 - nx*qz*C2*S1 + nz*qx*C2*S1 =d5*S3*S5 - d3*C3*C5 - a3*S3*S4*S5 - d5*C3*C4*C5 + d3*C4*S3*S5 + d4*C3*S4*S5,

Eq10 : ny*qx*C2 - nx*qy*C2 - ny*qz*C1*S2 + nz*qy*C1*S2 + nx*qz*S1*S2 - nz*qx*S1*S2=a2*S4*S5 - d5*C3*S5 - d3*C5*S3 + d4*S3*S4*S5 - d3*C3*C4*S5 - d5*C4*C5*S3 + a3*C3*S4*S5,

Eq11: nz*qy*S1 - ny*qz*S1 - nx*qz*C1 + nz*qx*C1 =a3*C5 + a2*C3*C5 - d4*C4*S5 - d5*C5*S - a2*C4*S3*S5,

Eq12: nz*qz2*S2 - nz*qy2*S2 - nz*qx2*S2 - nx*qx2*C1*C2 + nx*qy2*C1*C2 + nx*qz2*C1*C2 + ny*qx2*C2*S1 - ny*qy2*C2*S1 + ny*qz2*C2*S1 + 2*nx*qx*qz*S2 + 2*ny*qy*qz*S2 - 2*ny*qx*qy*C1*C2 - 2*nz*qx*qz*C1*C2 - 2*nx*qx*qy*C2*S1 - 2*nz*qy*qz*C2*S1 =a32*C5*S3 - a22*C5*S3 + d32*C5*S3 - d42*C5*S3 + d52*C5*S3 - 2*a2*d4*C5 - a22*C3*C4*S5 - a32*C3*C4*S5 + d32*C3*C4*S5 + d42*C3*C4*S5 + d52*C3*C4*S5 - 2*a3*d4*C3*C5 - 2*a2*a3*C4*S5 + 2*d3*d5*C3*S5 - 2*a2*d3*S4*S5 + 2*d3*d5*C4*C5*S3 + 2*d4*d5*C3*C5*S4 - 2*a3*d3*C3*S4*S5 - 2*a3*d4*C4*S3*S5 - 2*a3*d5*C5*S3*S4 - 2*d3*d4*S3*S4*S5,

Eq13: nz*qx2*C2 + nz*qy2*C2 - nz*qz2*C2 - nx*qx2*C1*S2 + nx*qy2*C1*S2 + nx*qz2*C1*S2 + ny*qx2*S1*S2 - ny*qy2*S1*S2 + ny*qz2*S1*S2 - 2*nx*qx*qz*C2 - 2*ny*qy*qz*C2 - 2*ny*qx*qy*C1*S2 - 2*nz*qx*qz*C1*S2 - 2*nx*qx*qy*S1*S2 - 2*nz*qy*qz*S1*S2=a22*C3*C5 + a32*C3*C5 + d32*C3*C5 - d42*C3*C5 + d52*C3*C5 + 2*a2*a3*C5 - a22*C4*S3*S5 + a32*C4*S3*S5 - d32*C4*S3*S5 - d42*C4*S3*S5 - d52*C4*S3*S5 - 2*a2*d4*C4*S5 + 2*a3*d4*C5*S3 - 2*a2*d5*C5*S4 - 2*d3*d5*S3*S5 + 2*d3*d5*C3*C4*C5 - 2*a3*d4*C3*C4*S5 - 2*a3*d5*C3*C5*S4 - 2*d3*d4*C3*S4*S5 - 2*d4*d5*C5*S3*S4 + 2*a3*d3*S3*S4*S5,

Eq14 : ny*qx2*C1 - ny*qy2*C1 + ny*qz2*C1 + nx*qx2*S1 - nx*qy2*S1 - nx*qz2*S1 - 2*nx*qx*qy*C1 - 2*nz*qy*qz*C1 + 2*ny*qx*qy*S1 + 2*nz*qx*qz*S1 =a22*S4*S5 + a32*S4*S5 - d32*S4*S5 + d42*S4*S5 + d52*S4*S5 - 2*d3*d4*C5 - 2*a3*d5*S5 - 2*d4*d5*C4*C5 - 2*a2*d3*C5*S3 - 2*a2*d5*C3*S5 - 2*a3*d3*C4*S5 - 2*a2*d3*C3*C4*S5 - 2*a2*d5*C4*C5*S3 + 2*a2*a3*C3*S4*S5 + 2*a2*d4*S3*S4*S5

5. Results

The software programme gives 8 possible solutions for various angles for given position of end effectors. These results are represented in **Error! Reference source not found.** below

i	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	45.0375	59.9968	134.999	30.021	-44.993	120.016
2	45.1454	63.519	130.74	31.399	-42.860	118.8291
3	1.3428	89.89	-92.515	-70.03	90.655	80.181
4	1.3428	89.89	-92.515	-70.03	90.655	80.181
5	57.575	117.61	87.481	62.02	-27.15	74.81
6	57.575	117.61	87.481	62.02	-27.15	74.81
7	29.22	63.9	45.158	-56.24	-21.021	125.74
8	51.548	70.053	38.951	63.31	-18.41	128.51

From the table, as expected, one of the solutions (set 1) matches the chosen values of angles in the direct kinematics.

Table 1 Data table: D-H parameters for IGM robot

i	αi-1(Degree)	ai-1(m)	di(m)	θi(Degree)
1	-90	0	0	θ1= 45
2	0	a2=0.432	0	θ2= 60
3	90	a3=0.019	d3=0.125	θ3 = 135
4	-90	0	d4=0.432	θ4= 30
5	90	0	d5=0.020	θ5= -45
6	0	0	0	θ6= 120

Conclusion

The elimination method used in this thesis can be used for any 6R manipulator with non-intersecting wrist as en end effectors. The inverse kinematic solution presented in this work can be used for further kinematic and dynamic analysis as shown below:

1. The workspace of a manipulator can be found by using the jacobian or continuation method.
2. The dynamic analysis of manipulator can be found using this solution.
3. The trajectory analysis can be done for the given workspace of the manipulator

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