



## Compressive Sensing Based Image Reconstruction

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### ABSTRACT :

Compressive sensing is a technique of image acquisition and reconstruction from a relatively fewer measurements than what the Nyquist theorem suggests; the sampling rate must be greater than twice the highest frequency in the signal for fidelity of image reconstruction. Compressive sensing is applicable when the signals under consideration are sparse. Two-dimensional discrete wavelet transform is applied for sparse representation of an image in this thesis. When sparsity is more, the performance of compressive sensing image reconstruction algorithm will be better. The sparse level of low frequency sub bands and high frequency sub bands are different. Two different compressive sensing measurement matrixes and recovery algorithms are used for the low-frequency sub bands and high-frequency sub bands for better results.

Medical field especially in MRI scanning, compressive sensing can be utilized for less scanning time, thus benefits patients. The reconstructed image will be better in both PSNR and visual quality

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### Keywords:

Compressive Sensing, Wavelet Transform, Discrete Cosine Transform, Sparsity, MRI.

## 1. Introduction

We are in the midst of a digital revolution that is driving the development and deployment of new kinds of sensing systems with ever-increasing fidelity and resolution. Signals, images, videos, and other data can be exactly recovered from a set of uniformly spaced samples taken at the so-called Nyquist rate. Unfortunately, in many important and emerging applications, the resulting Nyquist rate is so high that will end up with far too many samples. Alternatively, it may simply be too costly, or even physically impossible, to build devices capable of acquiring samples.

A new technology is required in this world where we are using HD videos, video conferencing, online games, etc. much more than ever. Now we have a solution, Compressive Sensing. The signal having a sparse representation can be recovered exactly from a small set of linear, non-adaptive measurements. It means it's possible to sense sparse signals by taking far fewer measurements, lower sampling rate, hence the name compressed sensing.

CS differs from classical sampling in three important respects. First, CS is a mathematical theory focused on measuring finite-dimensional vectors in  $\mathbb{R}^N$ . Second, CS systems typically acquire measurements in the form of inner products between the signal and more general test functions. Thirdly, the signal recovery is typically achieved using highly nonlinear methods.

In short, CS enables a potentially large reduction in the sampling and computation costs for sensing signals that have a sparse or compressible representation.

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**Nomenclature**

CS	Compressive Sensing
WT	Wavelet Transform
DCT	Discrete Cosine Transform
OMP	Orthogonal Matching Pursuit
DWT	Discrete Wavelet Transform
HD	High Definition
i.i.d	identically and independent dependent
MRI	Magnetic Resonance Imaging
MSE	Mean Square Error
PSNR	Pseudo SNR

In this paper we applied CS in the medical images and analyzed using different sparsity domain, measurement matrix and reconstruction algorithm. WT and DCT are used as sparsity domain. Hadamard matrix and Gaussian matrix are considered as measurement matrix. L1 minimization, Orthogonal Matching Pursuit and pseudo inverse multiplication method are used as recovery algorithm.

**2. Applications**

CS has already had notable impact on several applications. One example is medical imaging, where it has enabled speedups by a factor of seven in paediatric MRI while preserving diagnostic quality.

**2.1 Basic Concept of Medical Imaging**

CS is being actively pursued for medical imaging, particularly in MRI. Most MR images, like angiograms, have sparsity properties, in domains such as Fourier or wavelet basis. Generally, MRI is a costly and time consuming process. It is data collection process in which it dependent upon physiological and physical constraints. In addition, high gradient amplitudes and rapid switching can produce peripheral nerve stimulation. CS based techniques has been improved the quality of image through reduction in the number of collected measurements and by taking advantage of their sparsity, thus benefits patients.

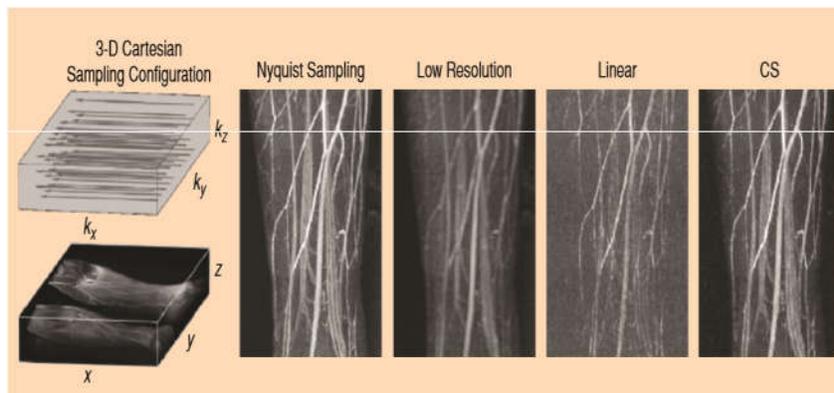


Fig 1: MRI image reconstructed using various techniques. [6]

### 3. Compressive Sensing – Basic Introduction

#### 3.1 Basic Block Diagram of CS based system

The recent theory of Compressive Sensing introduced by Candes, Romberg, and Tao and Donoho demonstrates that a signal  $K$ -sparse in one basis can be recovered from  $cK$  non-adaptive linear projections onto a second basis (call it the measurement basis) that is incoherent with the first, where  $c$  is a small over-measuring constant. When the measurement process is linear, the process of reconstruction is decidedly nonlinear.

Let  $x$  be a real valued, finite length, one dimensional, discrete time signal which is an  $N \times 1$  vector in  $\mathbb{R}^N$ . Thus by  $x \in \mathbb{R}^N$  can also be represented in orthonormal basis  $\Psi \in \mathbb{R}^{N \times N}$ .

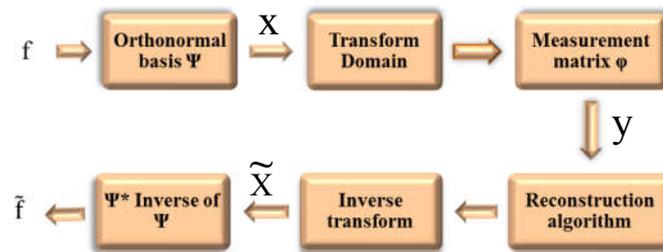


Fig 2: Block diagram of compressive sensing [1]

Thus signal  $x$  can be written as

$$x = \Psi f$$

- $f$  is  $N \times 1$  column vector of weighting coefficients  $f = (x, \Psi) = \Psi^T x$
- $f$  is the sparse representation of signal  $x$  in the orthonormal basis  $\Psi$ .

The signal  $x$  is  $K$ -sparse if it is a linear combination of only  $K$  nonzero basis vectors. When  $K \ll N$ , the signal  $x$  is compressible, means it has just a few large coefficients and many small coefficients.

We measure the signal  $x$  by sampling it with respect to a measurement matrix  $\Phi \in \mathbb{R}^{M \times N}$ . The measurement  $M \times N$  matrix  $\Phi$  must allow the reconstruction of the length- $N$  signal  $x$  from  $M < N$  measurements. The measurement process is not adaptive, meaning that  $\Phi$  is fixed and independent of signal  $x$ . Thus measurements  $M \times 1$  vector  $y$  can be represented:

$$y = \Phi \Psi f = A c s f = \Phi x$$

- $A c s = \Phi \Psi$  is the sensing matrix of  $M \times N$ .
- $\Phi$  is the measurement matrix.
- $\Psi$  and  $\Phi$  should be incoherent. [1]

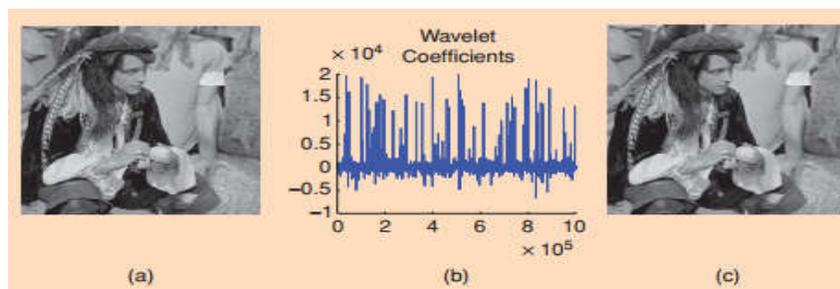


Fig 3: Sparsity and incoherence in wavelet transform enables the megapixel image to its approximation obtained by throwing away 97.5% of the coefficients with negligible perceptual loss. (a) Original Image (b) Wavelet transform coefficients (c) Reconstructed image [10]

### 3.2 Reconstruction Algorithms

#### Linear Optimisation

To recover the signal by l1-norm minimization; the reconstruction  $x^*$  is given by  $x^* = \Psi f^*$ , where  $f^*$  is the solution to the convex optimization program

$$(\|f\|_{\ell_1} := \sum_i |f_i|) \min_{f \in \mathbb{R}^N} \|f\|_{\ell_1} \text{ subject to } y_k = \langle \phi_k, \Psi f \rangle, \forall k \in M$$

That is, among all objects  $f = \Psi^{-1} x$  consistent with the data, pick only those whose coefficient sequence has minimal l1 norm. The CS theory uses L1 norm characteristics which are linear in nature and can be easily computed, thus offering a far simpler and faster way of estimating sparse signals from very limited number of measurements.

Greedy algorithm uses an iterative approach of the coefficient signal to the signal convergence is reached, or get an approximate increase of sparse signal in each iteration by calculating the measured data mismatch. OMP is one of greedy algorithms [7].

### 4. Proposed Algorithm

The proposed algorithm is given in fig 4. The image is undergone two - dimensional wavelet transform for more sparsity. In the image reconstruction based on the traditional compressive sensing algorithm, the same measurement matrix is used to measure the whole wavelet coefficients. However, since the high-frequency coefficients are sparse while the low frequency coefficients are not sparse [2], when putting the low frequency coefficients together with the high-frequency coefficients to multiply with the measurement matrix, the coherences among the low-frequency coefficients will be disrupted, which leads to a degraded performance of the reconstructed image [3]. The scaling coefficients of low-frequency component contain most of the image energy.

Due to these DWT features, two random CS sensing matrices are separately used for re-sampling the low-band and high-bands, which can be expressed as follows:

$$\begin{bmatrix} Y_L \\ Y_H \end{bmatrix} = \begin{bmatrix} \phi_L & \\ & \phi_H \end{bmatrix} \begin{bmatrix} X_L \\ X_H \end{bmatrix}$$

where  $Y_L$  and  $Y_H$  denote CS samples measured from low-bands and high-bands, while  $X_L$  and  $X_H$  represent the scaling and wavelet coefficients, respectively. At the decoder side, two different Compressive Sensing recovery algorithms are developed for the low-frequency high-frequency and also sub band, respectively.

The required number of CS measurements is much smaller than that of the DWT coefficients, and therefore, CS sampling. The challenge is to get the reconstructed image in better quality by adding more sparsity with maintaining incoherence. For this, we need to choose measurement matrix and recovery algorithm wisely.

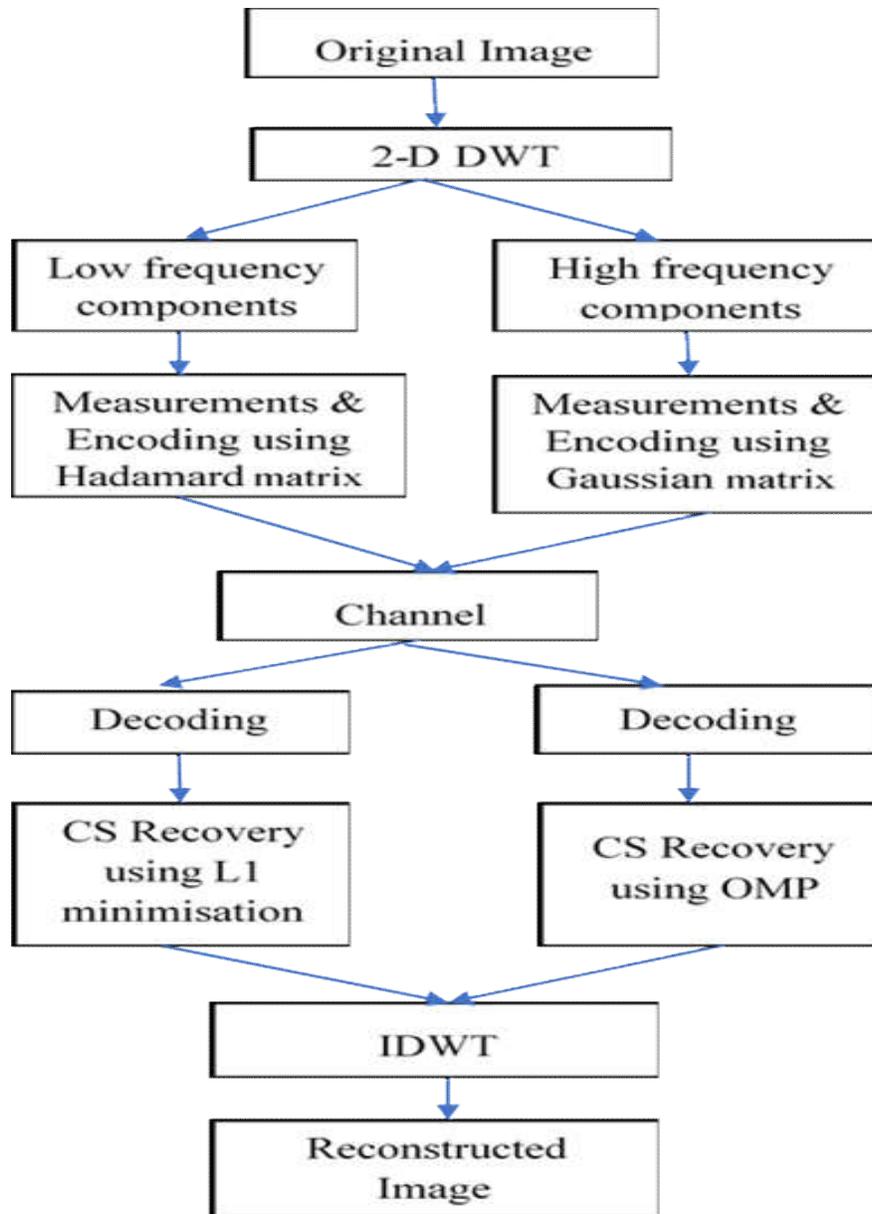


Fig 4: Proposed block diagram for implementation

## 5. Implementation

### 5.1 Algorithm Steps For Implementation

The steps of the algorithms are as follows:

- (1) Perform the wavelet transform of the  $N \times N$  image, and get the four wavelet sub-band coefficients  $\{LH_1, HL_1, HH_1, LL_1\}$ .

- (2) Use the measurement matrix to measure the three high-frequency sub-band coefficients  $LH_1$ ,  $HL_1$ ,  $HH_1$  to get the matrices of the measured coefficients while use another measurement matrix for the low-frequency sub-band coefficients  $LL_1$ .
- (3) Use the reconstruction algorithms to reconstruct the three high-frequency coefficients matrices  $\overline{LH}_1$ ,  $\overline{HL}_1$ ,  $\overline{HH}_1$ , and low-frequency coefficients matrix  $LL_1$ . Then together reconstruct the image.
- (4) The image reconstructed will be verified by its PSNR value.

### 5.2 Simulation setup

The scaling coefficients  $X_L$  are re-sampled by an i.i.d. random Hadamard matrix  $\Phi_L$ . The scaling coefficients  $X_H$  are re-sampled by an i.i.d. random Gaussian matrix  $\Phi_H$ . L1 minimization technique, OMP and Pseudo Inverse multiplication method reconstruction techniques are used as reconstruction algorithms.

We have used different size of images like 32x32, 64x64, 128x128 and 512x512. The transform method DCT is also compared with wavelet transform.

#### 5.2.1 Results of implementation of proposed block diagram.

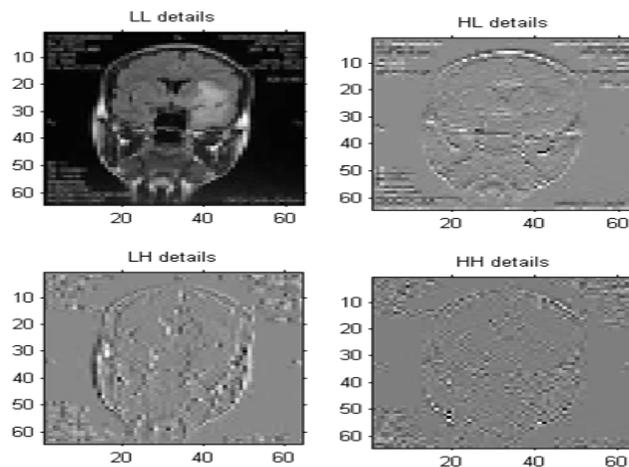


Fig. 5: Wavelet Transform Output of 128x128 image

The low frequency sub band LL has coarse information and details information in high frequency sub bands HL, LH, HH.

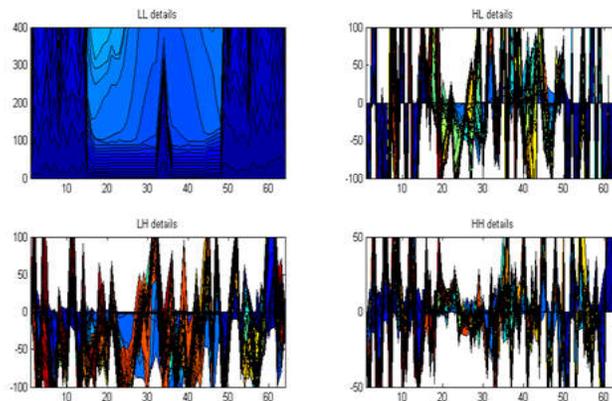


Fig. 6: Wavelet coefficients of 128x128 image before measurements taken.

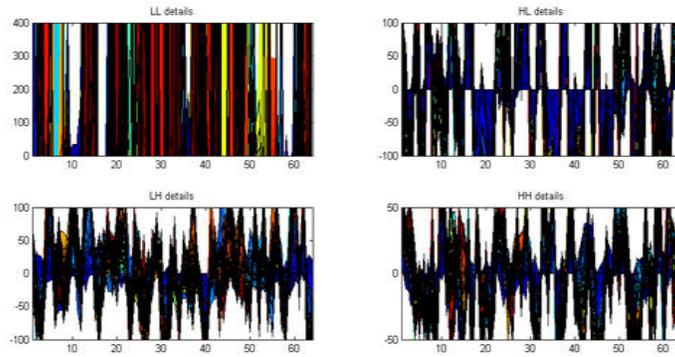


Fig. 7: Wavelet coefficients of 128x128 image after measurements taken.

CS makes the signal sparser, which is clear by comparing the Fig.6 and Fig.7. This sparsity leads to the efficient lossless compression. Due to more sparsity, the reconstructed image quality will be more. The outputs of different images are compared based on PSNR and MSE values with iteration value 50. The images are undergone WT.



Fig. 8: Different medical images [17]

Table 1: MSE and PSNR values comparison for different medical images for 50 iteration

		Medical image 1	Medical image 2	Medical image 3	Medical image 4
PSNR	L1 & PINV	43.83	54.85	31.76	30.11
	OMP	15.28	18.21	19.77	14.23
MSE	L1 & PINV	2.699	.212	43.27	63.36
	OMP	1971.3	981	684.86	2455

The outputs of different image sizes are compared based on PSNR and MSE values with iteration value 50. The image has undergone WT.

Table 2: MSE and PSNR values using L1 minimization or OMP with different sizes of images

		32x32	64x64	128x128	256x256	512x512
PSNR	L1 & PINV	-	50.08	42.74	33.31	31.46
	L1	-	50.08	42.74	-	-
	OMP	-	-	32.38	20.77	16.99
MSE	L1 & PINV	0	0.638	3.45	30.28	46.35
	L1	0	0.638	3.45	-	-
	OMP	0	0	610	542.17	1991.4

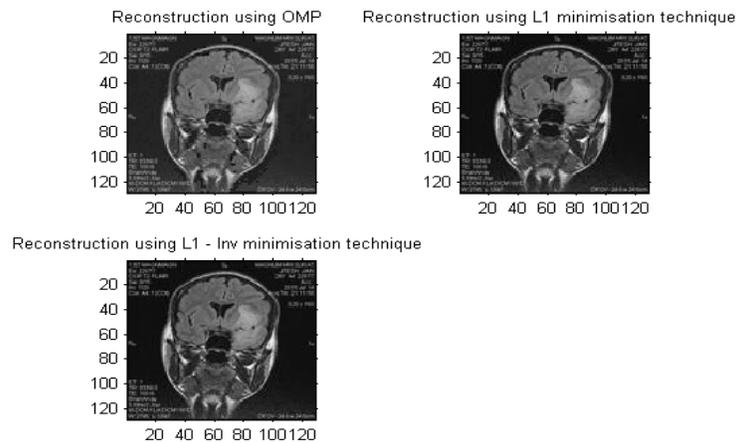


Fig. 9: Output of 128x128 image using various techniques

Table 3: PSNR comparison for various number of iterations

Iterations	PSNR using 32x32	PSNR using 64x64	PSNR using 128x128	PSNR using 256x256	PSNR using 512x512
5	17.87	15.76	14.53	13.5	12.96
10	23.37	18.57	15.86	14.6	13.66

15	36.67	21.08	17.25	15.34	14.14
25	-	30.39	20.25	16.84	15.1
50	-	-	30.39	20.77	16.99
100	-	-	-	33.88	21.12

The PSNR is compared with different number of iterations using OMP reconstruction algorithm.

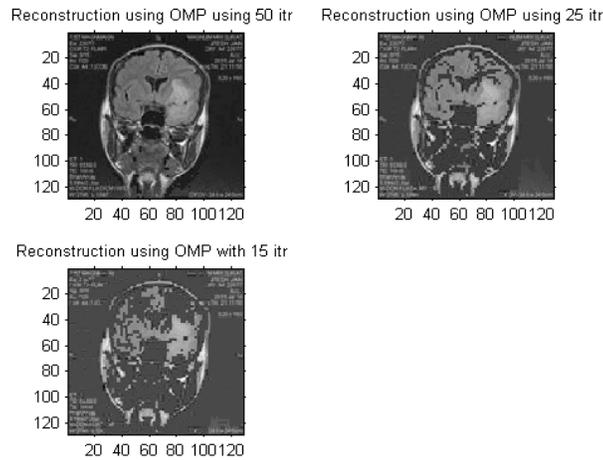


Fig. 10: Output of 128x128 image using different iterations of OMP.

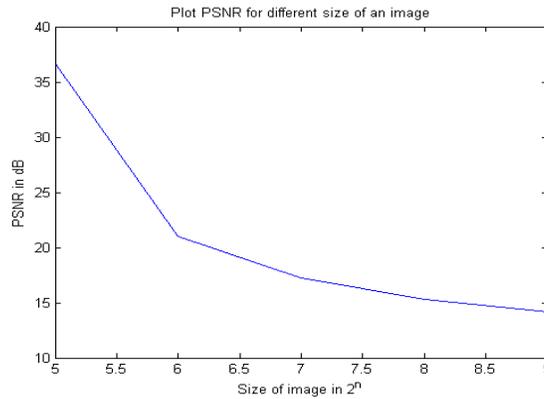


Fig 11: PSNR plot with different size of images.

PSNR values for different size of images respectively are plotted in fig 10. As size increases the PSNR values is getting decreased. But as size decreases the details of the image will get lost. As numbers of iterations are increasing, the image quality is also increasing as shown in fig 11. From table II we can conclude that less size and more iteration will result in better image.

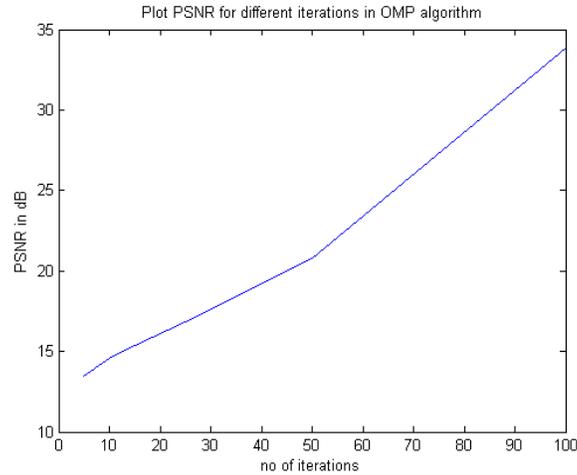


Fig. 12: PSNR for different iterations in OMP reconstruction algorithm.

The wavelet transform is compared with DCT. The image undergone DCT, and measurements are taken using Hadamard matrix and without packet loss in the channel. When size increases, the quality will be reducing. OMP with 50 iterations is used.

Table 4: MSE and PSNR comparison using DCT with Hadamard Matrix

		32x32	64x64	128x128	256x256	512x512
PSNR	OMP	-	50.18	38.59	28.55	27.14
MSE	OMP	0	0.546	9.1443	90.73	122.4

Table 5: MSE and PSNR comparison using DCT with Gaussian Matrix

		32x32	64x64	128x128	256x256	512x512
PSNR	PINV	19.9	19.45	19.06	18.04	18.02
	L1	36.13	28.45	26.02	22.24	20.35
	OMP	-	36.14	32.5	25.94	25.37
MSE	PINV	665.4	774.4	805.9	1013.5	1023.5
	L1	15.84	98.85	163.45	388.9	520.3
	OMP	0	15.16	36.56	165.4	188.6

OMP is better than L1 and PINV methods when Gaussian matrix is used as measurement matrix. Gaussian matrix represent random values.

**Inferences**

- When size of image increases, the PSNR value decreases.
- For small size images, OMP reconstruction is better. And for large size images, L1 minimization output is better.
- PSNR will increase when number of iterations is increased for OMP reconstruction algorithm.

- Hadamard matrix is better than Gaussian matrix when use after DCT.
- Wavelet transform is better when OMP reconstruction algorithm is used.
- Due to the sparsity in the signal, the required compression is achieved.

## Conclusion

The Compressive sensing measures a relatively small number of “random” linear combinations of the signal value. Sparsity, incoherence and nonlinear reconstruction are three main components of CS. The sparse nature of signals in a particular basis by taking measurements in an ‘incoherent’ basis is utilized in CS.

Wavelet transform is proven sparsity domain for many signals. The high-frequency coefficients are sparse while the low frequency coefficients are not sparse. So, both should be processed separately for better results. But in DCT transform, we cannot separate the components on basis of frequency. Hadamard measurement matrix is better. L1 minimisation is better with wavelet transform and OMP will be better with DCT.

In Medical field, CS will help in less number of samples, less radiation for patients. The medical images are inherently sparse and incoherence.

## Future works possible

- Reduce the processing time using Block-based CS method.
- Add enough sparsity by sparsity tuning and intra-prediction methods.

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